Study of a formalism modeling massive particles at the speed of light on a Machian basis

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(Dated: February 7, 2008)

In this paper we develop a formalism which models all massive particles as travelling at the speed of light(c). This is done by completing the 3-velocity **v** of a test particle to the speed of light by adding an auxiliary 3-velocity component **z** for the particle. According to the observations and laws of physics defined in our spacetime these vectors are generalized to domains and then two methods are developed to define c domain in terms of our spacetime(v domain). By using these methods the formalism is applied on relativistic quantum theory and general theory of relativity. From these, the relation between the formalism and Mach's principle is investigated. The ideas and formulae developed from application of the formalism on general relativity are compared with the characteristics of anomalous accelerations detected on Pioneer 10/11, Ulysses and Galileo spacecrafts and an explanation according to the formalism is suggested. Possible relationships between Mach's principle and the nondeterministic nature of the universe are also explored. In this study Mach's principle, on which current debate still continues, is expressed from an unconventional point of view, as a naturally arising consequence of the formalism, and the approaches are simplified accordingly.

PACS numbers: 03.30.+p, 03.65.-w, 03.75.-b, 04.20.Cv, 98.80.Hw

I. INTRODUCTION

It is a well known property of matter that a massive particle can not reach the speed of light. Particles that travel at the speed of light are observed to have no mass. Speed of light is a constant of the universe and a particle travelling at the speed of light is observed by all observers in spacetime travelling at this constant speed. Inspired by this constancy of speed of light we formed a theory which models all the matter in spacetime as matter travelling at the speed of light so that no frame of reference in the usual sense exists, all the particles have speed of light relative to each other irrespective of directions of their motion. Such a modelling satisfies and gives ideas and data encapsulating Mach's principle, when it is applied on relativistic quantum mechanics and general relativity; though no assumption in favor of Mach's principle is made during the formation of the theory. For this reason, before examining the details of the theory we will present versions of Mach's principle as interpreted by quantum theory and gravitation mainly referring to Feynman's discussions on this subject [1].

Mach's principle can be stated in different ways based on the approach to the phenomenon to be described. Regarding the motion of a particle, Mach's principle states that inertia represents the effects of interactions with faraway matter. According to Mach the concept of an absolute acceleration relative to absolute space was not meaningful, the motion of a particle should be described relative to distant matter in the universe. Similarly rotation should be rotation relative to something else, not absolute rotation. This definition of motion relative to distant matter may have significant implications on the motion of a test particle, since the usual mechanics assumes unaccelerated rectilinear motion to be the natural motion in the absence of the forces. Feynman noted in his Lectures on Gravitation [1] that "when accelerations are defined as accelerations relative to other objects, the path of a particle under no acceleration depends on the distributions of the other objects in space and the definitions of forces between objects would be altered as we change the distributions of other objects in space". In a similar manner he also argued that the absolute size of $g_{\mu\nu}$ which is 1 in Lorentzian metric may as well be different than 1, flat space may be $g_{00} = -g_{11} = -g_{22} = -g_{33} = \xi$ where ξ is a "meaningful number" not to be simply taken as 1. Feynman's ideas on Mach's principle show that Mach's principle may not only alter the laws of mechanics but also gives constants of motion some meaning.

On the quantum side, new fundamental units of length and time were definable with the development of quantum mechanics. These were the new absolutes of physics to be defined concerning Mach's principle. These absolute units of time and length are the factors that, for example, determine the maximum wavelength (or minimum frequency) of a photon that can create electron–positron pairs. Therefore, at each point of spacetime these absolute units must be defined as a natural measure of size and time. From Mach's viewpoint these natural measures are absolute only if they are compared to something else; i.e., distant matter in the universe. In other words, natural measures of size and time, such as Compton wavelength or the term $\hbar/mc^2$ are influenced and determined by distant matter in the universe. Ac-

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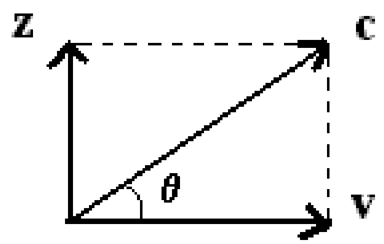

FIG. 1: visualization of the formalism as a sum of two orthogonal vectors.

cording to Mach these units are not absolute if they are not specified relative to something else.

Some results of Mach's principle stated above appear as a consequence of making all matter travel at the speed of light in the formalism which we develop in the next section.

II. GENERAL FORMALISM

We set the speed of a massive test particle moving with velocity $\mathbf{v}$ to the speed of light (c) by adding a nonoverlapping vector $\mathbf{z}$ that completes $\mathbf{v}$ to c in magnitude according to the simple expression

$$c^2 = \mathbf{v}^2 + \mathbf{z}^2 \tag{1}$$

where $\mathbf{v}$ and $\mathbf{z}$ are 3-vectors in position space. The geometric visualization of eq. (1) can be seen as a simple diagram in fig. (1).

To explain why such a visualization is selected, we should define constraints and properties loaded on these so-called vectors $\mathbf{v}$, $\mathbf{z}$, and $\mathbf{c}$. To accomplish this we let the characteristics of these vectors define their own domains instead of defining all vectors in one position space, where each domain has some properties compatible with observations and characteristics of each vector they belong to. The domains these vectors specify play a critical role in development of the theory.

$\mathbf{v}$ vector defines the velocity of the test particle measured in position space. One important property of the $\mathbf{v}$ vector is its measurability, in other words it specifies our local space as we observe in the vicinity of the test particle. We assume that the laws of physics in this domain are the same as those we define in our spacetime and neither particles with mass can travel at the speed of light nor such a massive object with speed c could be observed in this domain. Since all measurements can be done only in this domain and current laws of physics can not define equations of motion of a massive object travelling at speed c, all effects of c domain that are observable in the v domain are our only source of measurement and we propose that *an observer can observe the effects of any domain on a test particle only in the v domain by using the laws of physics defined in the v domain provided that any value (i.e., momentum, wave number) in that domain can be expressed in terms of values defined in v domain.* We will call this proposition *method* α for further reference. With this assumption we say that if z domain or c domain has observable effects on the particle and if these effects can be defined in terms of parameters in v domain, then we can measure these effects in our position space. But before using this assumption we should identify z domain in terms of v domain and since v domain and z domain form c domain, a relation defining c domain in terms of v and z domains is needed.

Major property of z domain we assign to it is that it is an unobservable domain and is totally unrelated from v domain so that no effect observable in z domain is directly present in the v domain. Due to this unrelatedness we represent $\mathbf{z}$ vector as an orthogonal vector to the $\mathbf{v}$ vector forming the simple diagram in fig. (1), which we use to develop and understand the formalism in a geometrical way. Since we can vary the parameters only in the v domain (i.e., $\mathbf{v}$ vector in the diagram), the range of z domain (i.e., range of its possible effects, represented by the magnitude of $\mathbf{z}$ vector in the diagram) can only be changed by varying the $\mathbf{v}$ vector while holding $\mathbf{c}$ constant in magnitude. We do not specify any more properties for the z domain, what entity it corresponds to in our space is an unknown to be found by using our formalism on the phenomena we will inspect later in this paper. But it is worth noting here the case that we do not even specify z domain to be local to the test particle and the $\mathbf{z}$ vector on the test particle even could be formed according to a nonlocal procedure.

Since a massive test particle travelling at the speed of light is not possible as observed in our spacetime(or equivalently in the v domain) and also there is no physical law that defines the dynamics of the massive particles at the speed of light, c domain is an unobservable domain but it is a combination of both v domain and z domain as the diagram suggests. The c domain is expected to include much more information than the v domain and z domain, therefore, once c domain is represented in terms of variables in v domain (our only observable domain in the vicinity of the particle) we will use the usual laws of physics to extract information according to our assumptions stated as method α. In the diagram the $\mathbf{c}$ vector is defined as a vector that has θ degrees between $\mathbf{v}$ vector and $90° - \theta$ degrees between $\mathbf{z}$ vector and this helps us to specify the magnitudes of $\mathbf{v}$ and $\mathbf{z}$ in terms of c, the speed of light. For the diagram we should also note that we can vary the magnitude of $\mathbf{v}$ vector between 0 and c, and magnitude being equal to c corresponds to the overlapping of the v domain with the c domain defining a massless particle travelling at the speed of light in our space, the only case c domain is totally observable and $\mathbf{z} = 0$. Thus θ has a range between 0° and 90°, 90° corresponding to the case that the massive particle stands still and $\mathbf{z} = \mathbf{c}$.

One last property which is common to all these do-

mains is the energy of the particle; in the c domain and z domain the energy of a particle has the same value as the energy measured in the v domain for that same particle. This should be valid, since no extra energy is introduced while completing the speed of the particle to the speed of light via addition of $\mathbf{z}$ vector.

The geometric diagram we presented in fig. (1) is very useful as far as velocity of the test particle is taken into consideration, but to generalize and develop the concepts of the formalism we make an abstraction by defining domains. This will enable us to propose a relation between the c domain and v domain so that variables such as momentum and wave number in c domain are definable in terms of variables in v domain, which in turn lets us inquire the effects of the formalism via quantum theory and some foundations of general relativity.

The relation between c domain and v domain

In order to use method α it is needed to formulate a relation between c domain and v domain so that c domain is interpretable in terms of v domain, where our measurements and laws of physics are guaranteed to be valid. To extract such a formulation we will first examine our diagram.

According to the diagram the magnitudes of the $\mathbf{z}$ vector and $\mathbf{v}$ vector are given as

$$\begin{aligned} z &= c\sin\theta, && (2)\\ v &= c\cos\theta, \qquad 0^\circ \leq \theta \leq 90^\circ && (3) \end{aligned}$$

where z and v are magnitudes of $\mathbf{z}$ and $\mathbf{v}$ vectors respectively. In v domain we can define the velocity of the test particle at point $\mathcal{P}$ as

$$\mathbf{v} = \frac{\partial \mathcal{P}}{\partial t}$$

where t is the time as measured by an observer in v domain. From special relativity the 3-momentum of this test particle in v domain is given as

$$\mathbf{p} = m\mathbf{u} \tag{4}$$

where $\mathbf{u}$ is the spatial part of the four velocity of the particle which can be defined in terms of spatial point $\mathcal{P}$ and proper time τ of the test particle given as

$$\mathbf{u} = \frac{\partial \mathcal{P}}{\partial \tau} = \frac{\partial \mathcal{P}}{\partial t}\frac{dt}{d\tau} = \mathbf{v}\frac{dt}{d\tau} \tag{5}$$

where

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} \tag{6}$$

for an observer in an inertial reference frame. Combining these results with the magnitude of $\mathbf{v}$ given in eq. (3) we get

$$p = \frac{mv}{\sqrt{1-\dfrac{v^2}{c^2}}} = mc\cot\theta \tag{7}$$

If the observer in v domain in some way manages to define $\mathbf{z}$ vector in terms of variables in v domain, we have

$$\mathbf{z} = \frac{\partial \mathcal{P}'}{\partial t}$$

where $\mathcal{P}'$ is the spatial point in v domain that corresponds to the effects of z domain. We can also define the spatial part $\mathbf{w}$ of the test particle due to z domain given as

$$\mathbf{w} = \frac{\partial \mathcal{P}'}{\partial \tau} = \frac{\partial \mathcal{P}'}{\partial t}\frac{dt}{d\tau} = \mathbf{z}\frac{dt}{d\tau} \tag{8}$$

where $\frac{dt}{d\tau}$ is the same as eq. (6), since all measurements are done in v domain according to method α. Implementing this result together with eq. (2) into eq. (4) we have

$$p_z = mw = mc \tag{9}$$

This means that whatever the magnitude of $\mathbf{z}$ is (even when $z = 0$ or $z = c$), the momentum associated with z domain according to method α is a constant.

Although we were able to express momentum in terms of velocity for z domain and v domain, we can not do such a momentum calculation for c domain in terms of speed of light and the magnitude of $\mathbf{v}$ vector as we did previously in eq. (9), since from our observations in v domain, particles travelling at the speed of light can have different values of momentum independent of their speeds. We must not contradict with our observations in the v domain if we want to construct a well defined theory. For that reason, we should develop a new formulation relating c domain to v domain so that we can apply method α.

One common property property of $\mathbf{v}$ vector and $\mathbf{z}$ vector is that at all cases their magnitudes are less than or equal to c. This enables c as being a constant in both domains to be a base magnitude in calculations of relativistic effects in terms of variables in v domain. Following this analogy, we propose that if we can find some *concept* which is not bounded by c at all we could use this as a base to observe the effects of c domain by using method α. The wave-particle duality of matter suggests a concept that satisfies this: the phase velocity of a particle is always bigger than or equal to c which is formulated as

$$s = \nu\lambda = \frac{c^2}{v} \tag{10}$$

where s is the phase velocity in v domain, ν is the frequency given by E/h, E being energy of the particle and λ is the de Broglie wavelength of the particle given as h/p. In terms of relativistic energy and momentum the phase

velocity is given as c^2/v and since maximum value that v can have in v domain is c, the minimum phase velocity is c. By using this property of matter waves and the general structure of the formalism developed so far, we define the relation between the c domain and v domain related to the same test particle through the equation

$$s_c^2 = s^2 + s_z^2 \tag{11}$$

where

$$\begin{aligned} s_c &= \nu\lambda_c, \\ s_z &= \nu\lambda_z, \\ s &= \nu\lambda \end{aligned}$$

are the phase velocities in the c, z, and v domains respectively.

Since we assume that the energy of the particle is same in all domains, eq. (11) can be simplified to

$$\lambda_c^2 = \lambda^2 + \lambda_z^2 \tag{12}$$

Here $\lambda_c = h/p_c$, $\lambda_z = h/p_z$, $\lambda = h/p$ and p_c, p_z, p are magnitudes of momenta of the same particle in c, z, and v domains respectively. It is clearly seen that eq. (12) enables us to represent p_c in terms of v domain once we specify p_z in terms of v domain. One thing we should consider for eq. (12) is the boundary conditions which must comply with our observations in the nature and the diagram for a consistent theory. We know from the diagram that as p goes to 0, $\mathbf{v}$ goes to 0 in magnitude and z domain overlaps with c domain which means that

$$\lambda_c = \lambda_z$$

for $v = 0$, $z = c$, and p vanishes leading λ to infinity.

For the case p goes to infinity v approaches to c and the diagram suggests the overlapping of v and c domains. But according to eq. (12) this is possible only if p_z vanishes so that λ_z goes to infinity and $\lambda_c = \lambda$ is satisfied. But from eq. (9) we showed that p_z is a constant nonzero value and for a massive test particle nonzero p_z even if $z = 0$ means that it can never reach the speed of light and c domain does not overlap with v domain completely. However, foundation of our formalism offers that as p goes to infinity v approaches very close to c and the diagram shows that c domain almost overlaps v domain completely and from these arguments we infer in the limit that

$$\lim_{p\to\infty} \lambda_c \to \lambda \text{ for a massive particle,} \tag{13}$$

$$\lambda_c = \lambda \text{ for a massless particle for any } p. \tag{14}$$

With this result we mean that for a massive particle the values of variables in c domain are very close to the values of variables defined in v domain as p goes to infinity so that we accept an overlap in the limit. This result will be used when we consider gravitation.

From the above discussion it is seen that constant p_z at any z is the cause that prevents a massive particle from reaching the speed of light. So this value defines a natural measure which constrains the speed of a massive particle in v domain always to a value under the speed of light and Mach's principle seems to appear here. But to investigate if there is a rigid connection between the formalism and Mach's principle, we need to identify the effects of z domain better while examining c domain by using method α.

This completes the development of the formalism, in the next two sections we will apply the formalism on the Dirac equation and general theory of relativity both to experiment it against well known facts and to use it to explain phenomena that can not be defined by current laws of physics.Through these sections we will use eq. (12) in the arranged form

$$p_c = \left(1 + \frac{p^2}{p_z^2}\right)^{-\frac{1}{2}} p \tag{15}$$

$$k_c = \left(1 + \frac{k^2}{k_z^2}\right)^{-\frac{1}{2}} k \tag{16}$$

where $p_c = \hbar k_c$, $p_z = \hbar k_z$, $p = \hbar k$, and k_c, k_z, k are the wave numbers of the particle in c, z, and v domains respectively.

We will refer to the concept eq. (12) stems from so frequently that we call it *method* β for simplicity.

III. APPLICATION OF THE FORMALISM TO THE DIRAC EQUATION

The Dirac equation is the relativistic version of the Schrödinger's equation where the hamiltonian is the linearized form of the relativistic energy–momentum equation given as

$$H = \sqrt{c^2P^2 + m^2c^4} = c\,\boldsymbol{\alpha}\cdot\mathbf{P} + \beta mc^2 \tag{17}$$

Here the 4×4 Dirac matrices $\boldsymbol{\alpha}$ and β are given in terms of Pauli and identity matrices as

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix} \quad \text{and} \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{18}$$

with

$$\boldsymbol{\sigma} = \sigma_1\mathbf{i} + \sigma_2\mathbf{j} + \sigma_3\mathbf{k} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\mathbf{i} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\mathbf{j} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\mathbf{k}$$

and

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The Dirac equation results in a multicomponent wave function which not only includes spin information but also defines "negative energy" solutions. When the electromagnetic interaction of a Dirac particle is taken into account, the Dirac equation successfully reduces in the

nonrelativistic limit to the Pauli equation for an electron [2] and the gyromagnetic ratio g automatically emerges as $g = 2$ which is very close to the correct value. Then we expect the formalism to be compatible with these results in the nonrelativistic limit and predict g as good as or better than the Dirac equation if it really describes the physical world in a true manner.

To check if the above conditions are satisfied by the formalism we will do our calculations in the nonrelativistic limit as well. For that reason, method β is used together with eq. (1) to define c domain in terms of v domain for $v \ll c$. When two sides of eq. (1) are multiplied by the square of the mass, we have

$$m^2c^2 = p^2 + p_z^2 \tag{19}$$

where we identify $p_z^2 = m^2z^2$ and $p^2 = m^2v^2$ in the limit $v \ll c$. Thus, the relation between z domain and v domain is satisfied and by inserting p_z into eq. (15) we have

$$p_c = \left(\sqrt{1 - \frac{p^2}{m^2c^2}}\right) p \tag{20}$$

So far p_c is considered in terms of magnitude of $\mathbf{p}$ but in applying the formalism on Dirac equation momentum operator will be taken as a 3–dimensional operator, so an expression for $\mathbf{p_c}$ whose magnitude satisfy eq. (20) is required. The most general representation in terms of corresponding momentum operators seems to be

$$\mathbf{P_c} = \left(\sqrt{1 - \frac{\mathbf{P}^2}{m^2c^2}}\right) \mathbf{P} \tag{21}$$

which does not prefer a predefined direction(as it might be imposed by $\mathbf{v}$ vector according to the diagram but generalization to domain concept helps us to use eq. (21) with no restriction).

Before inserting $\mathbf{P_c}$ into the energy-momentum equation and using the linearization procedure, we must linearize $\mathbf{P_c}$ so that the equation we have in the end is local in nature [2].For this reason the linearization first takes place according to the equation

$$1 - \frac{\mathbf{P}^2}{m^2c^2} = \left(\gamma - \frac{\boldsymbol{\eta} \cdot \mathbf{P}}{mc}\right)^2 \tag{22}$$

By comparing both sides of this equation it can be found that the following set of relations are satisfied by γ, $\boldsymbol{\eta}$ where γ and $\boldsymbol{\eta}$ are taken as constant matrices.

$$\begin{aligned} \gamma^2 &= 1, \\ [\gamma, \eta_i]_+ &= 0, \\ \eta_i^2 &= -1, \\ [\eta_i, \eta_j]_+ &= 0, \qquad i \neq j \quad i,j = 1,2,3 \end{aligned}$$

where $[\ ,\ ]_+$ defines the anticommutator relation.

Matrices satisfying these relations can be given in terms of Dirac matrices as

$$\gamma = \beta \qquad \boldsymbol{\eta} = i\boldsymbol{\alpha} \ .$$

Thus the linearized form of $\mathbf{P_c}$ is given as

$$\mathbf{P_c} = \left(\beta - i\frac{\boldsymbol{\alpha} \cdot \mathbf{P}}{mc}\right) \mathbf{P} \ . \tag{23}$$

Having linearized $\mathbf{P_c}$, it is now possible to apply method α by inserting $\mathbf{P_c}$ into the energy–momentum equation. Performing the linearization process for a second time according to the expression

$$H^2 = \left(c^2 \mathbf{P_c}^2 + m^2c^4\right) = \left(c\,\mathbf{C} \cdot \mathbf{P_c} + Dmc^2\right)^2 \tag{24}$$

results in a large set of relations between $\mathbf{C}$, D, γ and $\boldsymbol{\eta}$. The matrices $\mathbf{C}$ and D obeying these relations are found to be

$$\mathbf{C} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} \qquad and \qquad D = i\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \ .$$

With the derivation of $\mathbf{C}$ and D, all components of the method α applied Dirac equation are found and the electromagnetic interaction of an electron can be examined now. The final form of the equation is

$$\left(c\,\mathbf{C} \cdot \mathbf{P_c} + Dmc^2\right) \Psi(\mathbf{x}, t) = i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{x}, t) \tag{25}$$

To introduce the coupling of an external electromagnetic field to an electron the momentum operator in v domain is substituted by the momentum operator $\boldsymbol{\pi}$ given as

$$\boldsymbol{\pi} = \mathbf{P} - \frac{e}{c}\mathbf{A} \ . \tag{26}$$

Here $\mathbf{A}$ is 3-vector potential of the electromagnetic field. This $\boldsymbol{\pi}$ is substituted into $\mathbf{P_c}$ instead of $\mathbf{P}$ in the eq. (21).We also take the scalar potential ϕ of the field as $\phi = 0$ and as already mentioned, work to order $\frac{v^2}{c^2}$ to see the emerging of electron spin and gyromagnetic ratio from eq. (25). We will look for energy eigenstates given as

$$\Psi(\mathbf{x}, t) = \Psi e^{-iEt/\hbar} \tag{27}$$

where E is the eigenvalue of the Hamiltonian. The sizes of matrices $\mathbf{C}$ and D implies Ψ as a four component wave function and it can be grouped into two component spinors. This Ψ in terms of two component spinors can be written as

$$\Psi = \begin{pmatrix} \chi \\ \Phi \end{pmatrix} \tag{28}$$

where χ and Φ are two component spinors. By inserting eqs. (26), (27) and (28) into eq. (25) as specified and after some modification of the terms, eq. (25) becomes

$$\begin{bmatrix} E - c\,\boldsymbol{\sigma} \cdot \boldsymbol{\pi} & i\frac{\boldsymbol{\pi}^2}{m} - imc^2 \\ i\frac{\boldsymbol{\pi}^2}{m} + imc^2 & E + c\,\boldsymbol{\sigma} \cdot \boldsymbol{\pi} \end{bmatrix} \begin{pmatrix} \chi \\ \Phi \end{pmatrix} = 0 \tag{29}$$

By performing the matrix multiplication in eq. (29) we have a set of two equations which are

$$(E - c\,\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\,\chi + i\left(\frac{\boldsymbol{\pi}^2}{m} - mc^2\right)\Phi = 0 \tag{30a}$$

$$i\left(\frac{\boldsymbol{\pi}^2}{m} + mc^2\right)\chi + (E + c\,\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\,\Phi = 0 \tag{30b}$$

From eq. (30b)

$$\chi = i\frac{1}{mc^2\left(1+\dfrac{\boldsymbol{\pi}^2}{m^2c^2}\right)}(E + c\,\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\,\Phi \tag{31}$$

Here an operator in the denominator exists. This situation can be grasped as follows. For two operators A and B we have the relation [3]:

$$\begin{aligned}\frac{1}{A+B} &= A^{-1} - \frac{1}{A+B}BA^{-1}\\ &= \left(1 - A^{-1}B + A^{-1}BA^{-1}B + \cdots\right)A^{-1}\end{aligned} \tag{32}$$

In our case this relation can be interpreted as an expansion of Green's functions where A^{-1} is the Green's function of free particle (G_0) and B corresponds to the small interacting potential in terms of propagator theory(namely H_I). In terms of Green's functions eq. (32) can be written as

$$G = G_0 + GH_IG_0 \tag{33}$$

where

$$\begin{aligned}G_0 &= A^{-1} = \frac{1}{mc^2}\\ H_I &= -B = -\frac{\boldsymbol{\pi}^2}{m}\\ G &= \frac{1}{\left(mc^2 + \dfrac{\boldsymbol{\pi}^2}{m}\right)}\\ &= \frac{1}{mc^2}\left(1 - \frac{\boldsymbol{\pi}^2}{m^2c^2} + \left(-\frac{\boldsymbol{\pi}^2}{m^2c^2}\right)^2 + \cdots\right)\end{aligned} \tag{34}$$

Note that with natural emerging of such a propagator–like structure, the effects of virtual particles interacting with the particle itself are taken into account in the wave equation we develop; it means that the formalism introduces the basics of the propagator theory into Dirac equation. This would exactly help us to get information which was disjointly expressed before, by examining one equation. The reason starting from eq. (30b) is to see the effects of the total energy(i.e., $mc^2+\frac{\boldsymbol{\pi}^2}{m}$) as a propagator–like structure operating on the state Φ which seems to represent the negative energy part of the Dirac solution.

To get a full equation, eq. (34) is inserted into eq. (31) and eq. (31) is inserted into eq. (30a) resulting in

$$i\left(\frac{\boldsymbol{\pi}^2}{m} - mc^2\right)\Phi + i\,(E - c\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\,G\,(E + c\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\,\Phi = 0 \tag{35}$$

Evaluating the terms and multiplying both sides by 1/2 gives

$$\left(\frac{1}{2}EGE + \frac{1}{2}\,cEG(\boldsymbol{\sigma}\cdot\boldsymbol{\pi}) - \frac{1}{2}\,c(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})GE - \frac{1}{2}\,c^2(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})G(\boldsymbol{\sigma}\cdot\boldsymbol{\pi}) - \frac{1}{2}mc^2\left(1 - \frac{\boldsymbol{\pi}^2}{m^2c^2}\right)\right)\Phi = 0 \tag{36}$$

As seen above G handles all the situations: interaction of the total energy of the particle with the total energy itself, interaction of the total energy of the particle with the coupling electromagnetic field, interaction of the coupling electromagnetic field with the total energy of the particle and interaction of the coupling electromagnetic field with the field itself respectively. Since we work to

order $\frac{v^2}{c^2}$ we take

$$G = \frac{1}{mc^2}\left(1-\frac{\boldsymbol{\pi}^2}{m^2c^2}\right) \quad (37)$$

$$E = mc^2 + E_s \quad (38)$$

where E_s is the energy eigenvalue that appears in the Schrödinger equation.

We then insert eqs. (37), (38) into eq. (36) and after the cancellations are done we omit the terms that are of higher order than $\frac{v^2}{c^2}$(these are the terms including E_s^2, $E_s(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})$, $\boldsymbol{\pi}^2(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})$ etc.) and the resulting expression is

$$\left(\frac{\boldsymbol{\pi}^2}{2m} - \frac{e\hbar}{2mc}\,\boldsymbol{\sigma}\cdot\mathbf{B}\right)\Phi = 0 \quad (39)$$

where the identities

$$(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})(\boldsymbol{\sigma}\cdot\boldsymbol{\pi}) = \boldsymbol{\pi}\cdot\boldsymbol{\pi} + i\boldsymbol{\sigma}\cdot(\boldsymbol{\pi}\times\boldsymbol{\pi})$$

$$\boldsymbol{\pi}\times\boldsymbol{\pi} = i\frac{e\hbar}{c}\mathbf{B}$$

are used and $\mathbf{B}$ is the magnetic field.

It is clear that this equation is the Pauli equation and defines a particle with spin $\frac{1}{2}$ and gyromagnetic ratio $g = 2$. The same equation can be also derived for the χ component, and thus, in the classical approximation up to order $\frac{v^2}{c^2}$ the formalism is consistent with known facts. It can be also seen from eq. (36) that expansion of G to higher orders suggests that gyromagnetic ratio slightly differs from 2, which is due to a sum of powers of $\frac{\boldsymbol{\pi}}{mc}$, similar to approximation of the gyromagnetic ratio as the sum of powers of $\alpha \cong \frac{1}{137}$, the fine structure constant. This is compatible with experiment and the propagator theory since an electron can be as well presented in terms of one Dirac electron, a Dirac electron and a photon, a Dirac electron and several photons or several electron positron pairs etc. One thing we can extract from eq. (36) is that this equation also implies that spin is an intrinsic property of the particle. To show this we take $G = \frac{1}{mc^2} = G_0$ and put this into eq. (36). Taking $G = G_0$ means that no coupling has yet resulted in motion of the particle itself(i.e., no motion is observed in v domain). But it is a possibility that its surroundings (or virtual particles, maybe) affecting the state of the particle observed in v domain may interact with the field just before the particle does. In other words we do not constrain a simultaneity in the interacting times of v domain and z domain; it is probable that z domain can interact before the v domain (recall that we even do not underestimate the possibility of z domain being non-local in nature and operators in squareroots might as well lead to a nonlocal interpretation). If this is the case we simply insert G_0 into eq. (36) and after omitting the orders higher than $\frac{v^2}{c^2}$, it is seen that all other terms exactly cancel and we have for Φ(our z domain correspondent)

$$\left(-\frac{e\hbar}{2mc}\,\boldsymbol{\sigma}\cdot\mathbf{B}\right)\Phi = E_s\Phi \quad (40)$$

Thus eq. (40) shows that the interaction hamiltonian due to spin is active even if no momentum is present implying that spin is an intrinsic property of the particle. Moreover, it also shows that z domain introduces some measures and constants of size and time as Mach's principle suggests(This last result is very similar to the situation in eq. (9)). Another finding that can be inferred is that we can define the contents of z domain in terms of the virtual particles and interactions in the vicinity of the particle are taken into account, since an interaction just before the particle interacts with the field may result spin as an intrinsic property. But the definition of z domain should be a general one, true for all interactions so we move to gravitation in order to use the formalism to generalize the ideas presented so far, since both Mach's principle and spacetime have their most severe effects on gravitating bodies.

IV. FROM METRIC TO MACH'S PRINCIPLE

Now by using the assumptions and results established so far, we can develop a simple metric of spacetime to investigate the effects of the formalism according to general theory of relativity. The g_{00} component of such a metric can be developed by using method β and method α. By using the concepts of method β the eq. (12) can be modified to represent the c domain only in terms of v domain where the g_{00} component of the metric relating v domain to the c domain according to an observer in v domain can be given as

$$s_c^2 dt^2 = g_{00} s^2 dt^2 \quad (41)$$

Here v domain is taken as the base frame and the effects of z domain are embedded into the metric tensor g_{00} so that c domain is interpreted as a curved spacetime relative to the v domain. Here one novel thing is that we have used the phase velocity in the metric implied by the eq. (12) and in general by method β. And the measured time is the same in both domains for that same test particle and any resulting effects are loaded on the different values of the phase velocities. This enables us to calculate g_{00} in terms of phase velocities

$$g_{00} = \frac{s_c^2}{s^2} \quad (42)$$

But if the effects of this metric are observable in v domain, it should be observable according to the laws of physics and variables observed in v domain. We know that differences of phase velocities are not observed to cause relativistic effects in v domain, but time dilation is a well known effect of the spacetime depending on the gravitational potential at a point. Thus if such a metric in eq. (41) has observable effects, it should be observed as some type of time dilation and by taking the phase velocity in v domain as a base, eq. (41) can be restated as

$$s^2 d\tau^2 = g_{00} s^2 dt^2 \quad (43)$$

where τ is the proper time of the test particle that is due to the c domain. Since we speak of the c domain and the v domain associated with the same test particle we do not include spatial terms in this metric in order to examine only the relations between c and v domains. This metric does not mean that c domain curves the spacetime really but it rather causes effects on the test particle which seems to the observer as if extra gravitational forces are applied on it.

Some properties of g_{00} can be further investigated by using eq. (42). Given the phase velocities

$$s = \frac{\omega}{k} \qquad\qquad s_c = \frac{\omega}{k_c}$$

where ω is the angular frequency of the matter waves and since we assumed the energy has the same value in all domains, it is same in all of the expressions for a test particle. By inserting these into eq. (42) and from eq. (16) we have

$$g_{00} = \frac{s_c^2}{s^2} = \frac{k^2}{k_c^2} = 1 + \frac{k^2}{k_z^2} \tag{44}$$

So this shows that g_{00} *is a function of* k *only* and it does not depend on a single point in position space, and it is a constant in spacetime as long as k is constant.

Einstein stated in one of his communications with de Sitter [4] that, for complete relativization of inertia the metric should satisfy a set of boundary conditions, one of which is $g_{00}(\mathbf{x}) \to \infty^2$ as $|\mathbf{x}| \to \infty$. The $g_{00}(\mathbf{k})$ component defined in eq. (44) obeys this boundary condition as $|\mathbf{k}| \to \infty$ and the methods defined in the formalism introduces Machian ideas into general relativity.

According to the formulation of general relativity, an approximation to the Newtonian gravitational potential can be made in position space simply by using the metric of spacetime approaching the Minkowski metric at infinity (with $\eta_{00}(\mathbf{x}) = 1$ at $|\mathbf{x}| \to \infty$) in a static universe. In a static universe we define the spacetime metric by

$$c^2 d\tau^2 = \eta_{00}(\mathbf{x}) c^2 dt^2 + |d\mathbf{x}|^2 \tag{45}$$

From this metric the energy of a photon at a point $\mathbf{x}$ of the space can be given as below by defining the proper frequency ν_p as [5]

$$\nu_p = \frac{\nu}{\sqrt{\eta_{00}(\mathbf{x})}} \tag{46}$$

and the energy is given by

$$\varepsilon(\mathbf{x}) = \frac{h\nu}{\sqrt{\eta_{00}(\mathbf{x})}} \tag{47}$$

where

$$\lim_{|\mathbf{x}| \to \infty} \eta_{00}(\mathbf{x}) = 1$$

So the gravitational potential ϕ at $\mathbf{x}$ is given by

$$\phi(\mathbf{x}) = \frac{\varepsilon(\mathbf{x}) - \varepsilon(\infty)}{\varepsilon(\mathbf{x})} = 1 - \sqrt{\eta_{00}(\mathbf{x})} \tag{48}$$

which specifies an approximation to Newtonian gravitational potential for metric approaching Minkowski metric as $|\mathbf{x}| \to \infty$.

Similarly, in momentum space(namely in $\mathbf{k}$ space) a potential can be defined by using the definition of potential in position space. The energy we have according to eq. (43) is

$$\varepsilon(\mathbf{k}) = \frac{\hbar\omega}{\sqrt{g_{00}(\mathbf{k})}} \tag{49}$$

Now the value of $\varepsilon(\infty)$ should be found. In the expression in eq. (49) ω also goes to infinity as $|\mathbf{k}| \to \infty$ and this limit should be compatible with the discussion resulting in the eqs. (13) and (14). Therefore, in the limit $|\mathbf{k}| \to \infty$ the variables in c domain have values very close to the corresponding variables in v domain so that c domain almost overlaps with c domain resulting in

$$\lim_{|\mathbf{k}| \to \infty} \varepsilon(\mathbf{k}) = \hbar\omega \tag{50}$$

By using this we can define the potential in $\mathbf{k}$ space as

$$U(\mathbf{k}) = \frac{\varepsilon(\mathbf{k}) - \varepsilon(\infty)}{\varepsilon(\mathbf{k})} = 1 - \sqrt{g_{00}(\mathbf{k})} \tag{51}$$

Now we must define to what physical entity this potential corresponds. This value was derived by inserting the phase velocity into the metric by using the analogy in general relativity. What we have at hand is a potential depending on the wave number of a test particle and; since this potential depends on the wave number only, it is independent of changes in position space. So we should seek data or experiment results suspecting or detecting an effect of this kind, hoping that this helps us giving a meaning to this potential in $\mathbf{k}$ space. One candidate is the measurements of anomalous acceleration detected on the Pioneer 10/11, Ulysses and Galileo spacecrafts [6]. Especially the data on Pioneers are very exact. The anomalous accelerations of the Pioneers are measured as $\sim 8 \times 10^{-10}\, m/s^2$ directed toward the Sun and these values are measured to be fairly constant for a long time. Similar effects are measured for the Ulysses and Galileo spacecrafts with values of $\sim (12 \times 10^{-10} \pm 3)\, m/s^2$ and $\sim (8 \times 10^{-10} \pm 3)\, m/s^2$ respectively and they are also directed toward the Sun. The calculations Anderson et al. presented show that the data for Ulysses and Galileo are highly correlated with the solar radiation. But they also presented that it is still possible in principle for Ulysses to separate the solar radiation effects from the anomalous acceleration whereas for Galileo the measured acceleration does not seem to be reliable so we will only present the anomalous acceleration the formalism suggests for Galileo.

For velocities $v \ll c$, $\frac{k^2}{k_z^2} \approx \frac{v^2}{c^2}$ and Pioneer 10 is measured to have a constant speed of $12.2 \times 10^3\, m/s$ and Pioneer 11 is measured to have a similar velocity. Inserting this into eq. (51) by using the definition of g_{00} in eq.

(44) the potential gives a value

$$U(\mathbf{k}) = -8.27 \times 10^{-10} \tag{52}$$

which is very close to the values measured for Pioneers. In a similar manner we can calculate average speed of Ulysses which followed an elliptical orbit during the measurement of anomalous acceleration done by Anderson et al. By inserting the anomalous acceleration into eq. (51) and solving for v gives

$$v = 14.696 \times 10^3 \, m/s \; . \tag{53}$$

This value is only $194 \, m/s$ higher than the average value of the heliocentric speed measurements taken periodically during its voyage from 5.4 AU in February 1992 near Jupiter to perihelion at 1.3 AU in February 1995 [7].

And for the Galileo which has an approximately constant speed of $7.19 \times 10^3 \, m/s$ the $U(\mathbf{k})$ is calculated to be -2.8×10^{-10}. Compared to the value $(8 \pm 3) \times 10^{-10}$, this value is even smaller than the error part and due to high correlation of this value with solar radiation pressure, data from Galileo does not seem to be reliable.

From the above discussion it is seen that the potential $U(\mathbf{k})$ corresponds to the acceleration of a particle as a physical variable. The characteristics of this acceleration is that it is defined in terms of $\mathbf{k}$ and independent of the position. Also the potential referring to an acceleration magnitude does not explicitly specify a direction for the acceleration but $g_{00}(\mathbf{k})$ being always greater than 1 suggests that it is always negative (i.e., opposes motion).

The success of potential $U(\mathbf{k})$ in defining the magnitudes of anomalous accelerations of Pioneers and Ulysses suggests that this anomalous acceleration is due to Mach's principle which can be stated for this case as inertial properties of objects are determined according to distant matter in the universe. For the Pioneers' trajectories, which are heading out of the Solar system and far enough from any strongly gravitating system, the inertia of the spacecrafts respond differently to their motion by opposing it due to Mach's principle. Ulysses' trajectory, which was an elliptical orbit during the measurement of the anomalous acceleration, also admits the intuition that its inertia responds to the elliptical motion slightly differently and this results as an anomalous acceleration that can not be interpreted in terms of other gravitating bodies close to the spacecraft. In other words, while the gravitational interactions make Ulysses travel in an elliptical path, the resulting elliptical motion makes inertia of Ulysses respond differently. This interpretation may suggest inequivalence of inertial and gravitational masses, but rather than accounting this anomaly to the difference of inertial and gravitational masses we account this to a direct effect of distant(in fact all) matter in the universe, depending only on the motion of the particle according to eq. (51). We are led to assume in this way because there is an apparent distinction between these spacecrafts and other celestial bodies such as planets in the universe. During their journey, these spacecrafts do maneuvers and even change their spin independent of the pure gravitational attraction between the spacecrafts and the Solar system. And though Ulysses is in an elliptical orbit, its motion is not completely governed by the Sun and other planets, whereas the other celestial bodies orbiting around the Sun have orbits determined by Sun and other planets, which make these orbiting celestial bodies bound to some conditions mainly determined by gravitation. But Ulysses' orbit is not a bound orbit totally, the spacecraft can maneuver when needed independent of the Sun's interaction with the spacecraft, making it impossible to assign boundaries to all its motion only in terms of its gravitational interaction with the Sun or other celestial bodies. These differences could result in significant inertial effects and the only boundary that can be used in modelling the motion of such a particle may be the size of the universe or even infinity. In fact our potential $U(\mathbf{k})$ helps us to answer how all the matter in the universe can affect an object. This is due to the fact that if $U(\mathbf{k})$ is a function of $\mathbf{k}$ only then it is independent of position space and if this $U(\mathbf{k})$ defines a constant value which we can measure in position space (which is the case), we should be able to define a relation between $U(\mathbf{k})$ and the position space. One obvious connection between $U(\mathbf{k})$ and position space can be the sum of fourier transforms of the potentials in the position space, given as

$$U(\mathbf{k}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \phi(\mathbf{x}) \, e^{-i\mathbf{k}\cdot\mathbf{x}} \, d^3\mathbf{x} \tag{54}$$

where

$$\phi(\mathbf{x}) = \phi_1(\mathbf{x}) + \phi_2(\mathbf{x}) + \phi_3(\mathbf{x}) + \cdots$$

is the sum of potentials and energy per unit mass terms that affect the motion of the test particle at a point $\mathbf{x}$, resulting from all matter and energy in the position space.

Eq. (54) defines $U(\mathbf{k})$ as superposition of potentials in the position space which directly refers to the statement that the matter in the universe affects the inertia of the object, namely Mach's principle. This equation also explains why $U(\mathbf{k})$ is constant since for any $\mathbf{k}$ all the contributions of the potentials in position space are included so that nothing is left in position space to change this $U(\mathbf{k})$.

We have defined a value in $\mathbf{k}$ space as superposition of its corresponding values in position space. We can backtrack what we have done in assuming eq. (54), we can replace $\mathbf{k}$ in $U(\mathbf{k})$ with $\mathbf{x}$ on the left hand side and defining a potential as $U(\mathbf{x})$ means removing the superposition of the potentials on the right hand side, which results in

$$U(\mathbf{x}) = \phi_1(\mathbf{x}) + \phi_2(\mathbf{x}) + \phi_3(\mathbf{x}) + \cdots \tag{55}$$

where superposition is eliminated by removing the fourier transform.

Here the terms that have sources close to the particle will survive so that the effects of the faraway matter will be negligible as it is the case in position space.

Implications above arise completely from the relationships between momentum and position spaces. In studies investigating the effects of distant matter where position space is considered only, the effects of distant matter show up by expressions implying variations in values of constants of the universe. In this way, the effects are valid in every point of space and independent of position. In our formalism, when applied to general relativity, the effects(as we take them as effects of distant matter) including any change in values of basic constants arise due to the relationships between the position space and momentum space whose variables change independent of position, instead of a direct change in values of constants of the universe. Sciama [8] showed in his study on the origin of inertia that the scalar gravitational potential of the whole universe satisfies the condition

$$G\Phi = -\,c^2$$

where $\Phi = -2\pi c^2 \rho \tau^2$ and ρ is the mean density of the matter in space and τ is a Hubble law related constant. This equation relates the value of the gravitational constant G to the gravitational potential of the universe. This result in fact is a consequence of many general relativistic models of the universe and Feynman [1] uses this expression to calculate the effects of distant matter on the term g_{00} in the vicinity of a star with mass m giving (with $\hbar = c = 1$)

$$g_{00} = 1 + \frac{2Gm}{r}$$

This expression is what our formalism suggests when $\frac{k^2}{k_z^2} \approx \frac{v^2}{c^2}$ and $v^2 = \frac{2Gm}{r}$. It must be again pointed out here that the formalism applied to general relativity gives the effect (acceleration) not the source (potential) when the tensor element g_{00} is depending on momentum space variables, where we were expecting a potential value in fact. Therefore this effect can be defined as fictitious and source is defined as the distant matter (a nonlocalized entity) according to the relationships between momentum and position space. From point of view of the formalism, this situation can be explained as a source in c domain which is affecting v domain where we can not detect the source as a localized point in v domain since c domain and thus the source have an unobservable part in it, which is z domain. Such an approach of the formalism also introduces inspection of the variations and uncertainties in scales of measurements performed on observables of a system in terms of the relationships between momentum and position spaces.

Turning back to $U(\mathbf{k})$ again, we see another important situation when the sum of the potentials $\phi(\mathbf{x})$ on a test particle at a point $\mathbf{x}$ is constant. In this case we have

$$U(\mathbf{k}) = \phi\, \delta(\mathbf{k}) \tag{56}$$

where $\delta(\mathbf{k})$ is the Dirac delta function and ϕ is a constant value.

Therefore, $U(\mathbf{k})$ is zero unless $\mathbf{k} = 0$ for $\phi(\mathbf{x})$ being constant. $\phi(\mathbf{x})$ constant means that energy is either *conserved* or it does not *depend* on $\mathbf{x}$ at all. For gravitating bodies orbiting around another gravitating body, the total energy of the system is conserved and $U(\mathbf{k}) = 0$ is satisfied, the anomalous accelerations observed for Pioneers will not be observed for orbiting bodies. This gives an explanation for not detecting the same effects for the planets in the Solar system, which should have been observed till this time if they were present. For quantized systems where energy is quantized and does not depend on position, the total energy is also constant and we again have $U(\mathbf{k}) = 0$ unless $\mathbf{k} = 0$. Having nonzero values only for $\mathbf{k} = 0$ further introduces the concept of uncertainty in quantum mechanics, what this last result might mean in terms of quantum theory is discussed below.

Since we established such a relation between $\mathbf{k}$ space, position space and Mach's principle, we can use principles of quantum mechanics to define further characteristics of the universe according to the formalism.

From quantum mechanics it is a postulate that to every observable there is an operator which is hermitian. Therefore, we can define eq. (51) as an operator and since we can linearize it as done for a similar expression in the previous section, we see that this operator is in fact linearized in terms of momentum operator. We can also define eq. (51) as a nonlocalized operator by power expanding it in powers of momentum operator(taking $\mathbf{P_z}$ as constant):

$$\begin{aligned} 1 - \sqrt{g_{00}(\mathbf{P})} &= 1 - \sqrt{1 + \frac{\mathbf{P}^2}{\mathbf{P_z}^2}} \\ &= 1 - \left(1 + \frac{1}{2}\frac{\mathbf{P}^2}{\mathbf{P_z}^2} - \frac{1}{8}\frac{\mathbf{P}^4}{\mathbf{P_z}^4} + \cdots\right) \end{aligned}$$

Quantum mechanics also states that for each operator, its observables are the eigenvalues of that operator which must be real. It is a well known issue that for every free particle state we can define an eigenvalue which is $\hbar\mathbf{k}$ for momentum operator but for bound particles there are no eigenvalues for the momentum operator satisfying the boundary conditions [9].This situation, during the development of the quantum theory in its early times, made the first significant difference of the quantum theory from classical theory that the expected value should be used and determinism was lost. Since the operator $1 - \sqrt{g_{00}(\mathbf{P})}$ is also dependent on the momentum operator, there is also no eigenvalue for this operator satisfying the boundary conditions for a bound particle. In terms of the diagram in fig. (1), assigning boundary conditions means limiting the allowed values in v domain, this in turn means limiting the z domain which introduced the effects of the whole universe. z domain being limited only to some permissible values means that one can not introduce all the effects of the matter in the universe since boundary conditions in v domain allow only some selected part of the points in the universe to interact with the bounded system. This means that $U(\mathbf{k})$ is no

more constant and does not cover the effects of all matter in the universe for that system, the excluded part of the universe may as well affect $U(\mathbf{k})$ and change the constant measures of size and time it specifies. This situation can be also interpreted in such a way that for a bounded system each point in the universe defines its own constants of measures of size and time so that measurements taken at different points in spacetime *measures the events according to different scales* (e.g., each point in spacetime may define different Compton wavelengths). This means that determinism is lost, we have no absolutes of time and size same for all points in the universe valid for all time instants and we can not fully predict future dynamics of a system depending on its past dynamics.

This situation could as well be generalized to include all other bound objects. These bound objects also constrain the v domain and z domain and this makes the constantness of $U(\mathbf{k})$ be destroyed since there is still matter that does not satisfy the bounds both in v domain and in z domain and only the expected values are measured as a result.

This is an expected characteristic of our universe indeed, elementary particles and fields seem to consist of quanta and these quanta form bounded systems which are also quantized due to their building blocks and for such a system one can not make the whole universe to satisfy the boundary conditions of the bounded system so that we can not define constant $U(\mathbf{k})$ or other measures of size and time valid for all points. So this also answers the questions why $U(\mathbf{k})$ is observable in our space if the observed particle is free and why we don't say that equivalence principle is violated for the Ulysses or Pioneer case. In fact equivalence principle applies only to experiments that are isolated from the rest of the universe, otherwise it results in paradoxes [4]. Moreover, these discussions suggest an answer to the question what entity z domain corresponds to, it seems that z domain corresponds to the rest of the universe(or position space in general) that interacts with the test particle.

One more implication of the formalism is due to the phase velocity we used to establish these ideas. If we accept that the effects presented in this study are propagating with the phase velocity which is always larger than c for matter waves, this enables us to define Lorentz frames where a particle can move on straight lines free of any gravitational effects of other bodies or stand still, and in all these cases Mach's principle stating that inertial frames are determined relative to distant stars can be satisfied. For distant matter the phase velocity carries the effects of inertia faster than light speed and before the causality between a free test particle and distant matter occurs, the inertial properties of the mass can be already transmitted. Thus, until the causality is established(we assume that gravitons also travel at the speed of light) the free particle is in a Lorentz frame where all inertial effects are seen. Note that since $g_{00}(\mathbf{k})$ is independent of position space we can further assume that the Lorentz frame has $g_{00} = 1$ when $\mathbf{k} = 0$ and $g_{00}(\mathbf{k}) > 1$ for free particle in motion.

V. CONCLUSIONS

In this paper we developed a formalism which models all particles travelling at the speed of light and then applied it on two distinct concepts, namely the quantum theory and general relativity. In doing these, it has been seen that the concept of phase velocity and wave-particle duality of matter played a critical role.

One important implication of the formalism is that it includes the Machian viewpoint, and embeds Mach's principle in other theories via the contribution of z domain into our space, when it is applied on these theories. For general relativity, the formalism results in ideas that were developed by several physicists including Einstein, but the formalism approaches these ideas differently. Einstein spent much effort to make his theory of general relativity Machian [4], he proposed to develop a Machian theory by imposing boundary conditions to get rid of non-Machian solutions of general relativity and following these ideas he was led to the idea of a *closed* universe to make the theory Machian. And Einstein's theory has the concept of Minkowski metric as a valid solution which is totally non-Machian in character.

The formalism faces these problems as well, boundary conditions became an important case that should be taken care of. The formalism handles the situation as a special case of the core principle stating that matter here is governed by matter there, such that the scales determined by distant matter for the bounded system are not nonchanging scales valid at every point of the four dimensional manifold and only a selected set of points in space time have a common scale, but rest of the points in spacetime define their own scale leading to measurements being statistical in nature. Therefore, Mach's principle is not violated, matter still depends on other matter in spacetime, but this relativeness is no more a strict distinction between the matter and the rest of the universe; the matter could also have the concept of relativeness defined differently by different points in the universe leading a way to the so-called loss of determinism. Once this is accepted, there is no need to impose conditions such as being closed and finite for the universe from a Machian viewpoint. Moreover, the inclusion of phase velocity in the formalism also enables a procedure for the construction of a Minkowski metric where all inertial effects due to the distant matter are established but no gravitational effect of that distant matter has been applied yet.

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